Efficiency of Human Power Production by Vertical Pumping

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Mechanical efficiencies are calculated for a human doing work in a standing and stooping cycle while on a movable platform. An unsteady force is generated which does useful work in oscillating the platform on its suspension system. Such a vertical pumping motion has been proposed for a man-powered ornithopter. The theorem of virtual work provides the efficiency expression. Analog simulation reveals that square wave force exitation is more efficient than sinusoidal or triangular. Design curves show some unexpected requirements for matching man and machine, and very poor efficiency if care is not taken. Losses are due to gravity and human inability to store energy in unloading portions of the cycle. A spring-dashpot suspension allows efficiencies of up to 88% in cases involving sinusoidal exitation. If the spring force is set to zero, one obtains a maximum of 64% efficiency for harmonic exitation. Some improvement can be made by adding toe straps to the human and/or by forcing the cage in a square wave. The novel feature, making this work differ from ordinary vibration work, is the switching logic needed to distinguish loading and unloading portions of the cycle.

Nomenclature

B	= half-amplitude of nondimensional leg extension
c	= equivalent viscous damping coefficient of suspension
F	= half-amplitude of leg force
$\mathfrak{F}(x,\dot{x},\ddot{x})$	= external force on cage
$f(t)$	= dimensionless periodic function
k	= spring constant of suspension
M	= cage mass
m	= human mass
P	= power
S	= half amplitude of leg stroke
T	= period of oscillation
W	= work
$x(t)$	= platform position
$y(t)$	= human c.g. position
$z(t)$	= arbitrary function
α	= phase lag, Eq. (16)
δW	= virtual work
δW_{hum}	= internal work by human during virtual displacement
ϵ	= biological penalty factor
ζ	= damping ratio, $c/[2(kM)^{1/2}]$
η	= efficiency
ϕ	= phase lag, Eq. (12)
ω	= circular frequency
ω_n	= natural frequency of cage alone $(k/M)^{1/2}$

Introduction

HUMANS use rotary and receiprocating cycles of motion in many work and play situations including rowing boats, riding bicycles, and bouncing on trampolines. Power production and efficiency of certain kinds of motion have been studied carefully, especially the rotary motion used in pedalling a bicycle or flying a man-powered plane, and particularly in the British literature.[1,2] In 1972, Smith proposed a rigid wing ornithopter propelled by a pilot moving in a standing and stooping cycle within the fuselage.[3] It was believed that this is a highly efficient method of transmitting energy to the fuselage and then to the airstream. The question of how efficiently energy can be transmitted from pilot to fuselage is by no means trivial, however, and may be the critical part of the power cycle.

A study is made of the efficiency and power generation of a human moving in a vertical, closed cycle of motion (Fig. 1). The direct application is to man-powered flight, but a general approach will be taken in terms of the suspension of the platform so that other man/machine situations can be covered. The platform position $x(t)$ and the human's c.g. $y(t)$ are measured from an inertial frame. The external force on the cage is found from the aircraft's stability derivatives, or from a given suspension system.

The novel factor that makes this problem different from a standard vibration problem is that the human does work in a irreversible way. He cannot store energy on the portions of the cycle during which the platform and gravity do work on him (the so called "unloading" portion). The platform and gravity can accelerate the human and create kinetic energy, but any portion of a leg stroke in which platform force and gravity tend to further this stroke must release energy in heat. One must be careful in analyzing the cycle, then, to not consider this component of work as useful mechanical energy.

Gravity can cause a loss of efficiency, because it creates a dead weight that must be cycled through all motion. Gravity never does any net work over a closed cycle, but provides a bias force, which does affect the division of the cycle into loading and unloading portions. Physiologists and engineers have realized that gravity represents a loss term in human motion, such as running a race, and can account for the portion of energy lost when a vertical cyclic motion is performed on a rigid surface.[1] Gravity's role in the man/platform combination is more subtle, but still must be regarded as the chief culprit in the energy loss.

Equations of Motion

Assume that the mass of the human is concentrated at his c.g., and that the loading passes entirely through his legs.

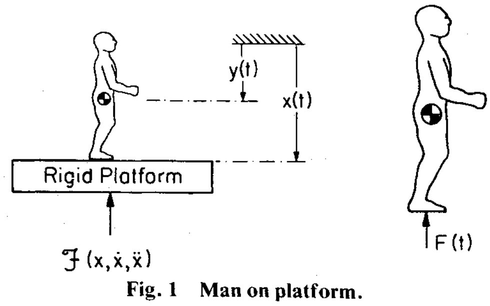

Fig. 1 Man on platform.

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(The analysis also is valid if hand supports are available, but is conceptually easier without them.) The motion studied here is periodic.

Forces due to cage suspension, whether mechanical or an aerodynamic equivalent, will be $\mathfrak{F}(x,\dot{x},\ddot{x}) = c\dot{x}+kx$. Any apparent mass effects due to fluid forces on the platform can be included in the platform inertial term $M\ddot{x}$. The force $F(t)$ in the human leg is defined positive in compression and has a gravitational bias $F(t) = Ff(t) + mg$, where $f(t)$ is periodic with half amplitude of unity and F is the force amplitude. The coordinates x and y are defined so that the leg extension $x-y$ is zero when the human is half crouched (Fig. 2). The equations of motion are simply

$$m\ddot{y} = -Ff(t) \tag{1}$$

$$M\ddot{x}+c\dot{x}+kx = Ff(t) + (M+m)g \tag{2}$$

The steady-state, oscillatory solutions to these equations are found easily.

Efficiency

The system efficiency will be defined at first in purely mechanical terms. Efficiency is the ratio of useful mechanical work done by the human on the platform to the total mechanical work done by the human. Work is defined in the inertial frame. The useful work done by the man on the platform per cycle is

$$W_{\text{useful}} = \int_0^T F(t)\dot{x}(t)\,dt \tag{3}$$

One might be tempted to include as useful work that which the man does against gravity, on the grounds that it will be returned to the platform at some later time in the cycle. This is fallacious, because all work done by the man on the platform must be accomplished through his legs (his only point of contact) and is already included in Eq. (3).

The total mechanical work done by the man can be seen best by applying the principle of virtual work and D'Alembert's principle. Consider the human, acted upon by gravity and the cage, during a virtual displacement

$$\delta W = \delta W_{\text{hum}} + mg\ \delta y - m\ddot{y}\ \delta y - F(t)\delta x = 0 \tag{4}$$

The internal work δW_{hum} must be supplied to maintain the energy balance at any instant. It does not include any wasted energy (e.g., blood circulation, flapping arms) which does not contribute to vertical equilibrium. Including force equilibrium, $F(t) = mg - m\ddot{y}$, one obtains

$$\delta W_{\text{hum}} = (mg - m\ddot{y})\ \ (\delta x - \delta y) \tag{5}$$

In retrospect, this energy argument gives the correct results, since the work increment (as seen by the human) is logically the force in his legs times the extension of his legs. Note that this work is done against gravity as well as against the platform.

There is a problem of interpretation if the increment of work in Eq. (5) is negative. The loading and unloading portions of the cycle depend on the signs of the factors as shown in Table 1.

Table 1 Relation of signs of factors to loading and unloading

$(mg-m\ddot{y})$	$(\delta x-\delta y)$	case
+	+	loading
+	−	unloading
−	+	unloading
−	−	loading

The latter two cases in Table 1 correspond to tension in the legs and are obtainable only if straps hold the feet to the floor. The human does not absorb energy during unloading portions of the cycle, but rejects it as heat. In addition, when comparing this reciprocating cycle with a rotary pedalling cycle (which usually has no unloading stroke), one might want to include a biological penalty factor for making the human pass through an unloading portion of a cycle. This could be done by defining the total human work on a cycle as

$$W_{\text{hum}} = \int_0^T \left| F(t)\ \ (\dot{x}-\dot{y}) \right| dt = \int_0^T \left| (mg-m\ddot{y})\ \ (\dot{x}-\dot{y}) \right| dt \tag{6}$$

where the curly brackets are defined for an arbitrary function $z(t)$

$$\left| z(t) \right| \equiv \begin{cases} z(t) & \text{when } z(t) \geq 0 \\ \epsilon\, |z(t)| & \text{when } z(t) < 0 \end{cases} \tag{7}$$

A choice of biological factor $\epsilon = 0.33$, say, then would allow a fairer comparison between the current cycle and a rotary cycle. In the long run, however, human experiments should be run on the vertical reciprocating cycle to find the true physiological penalty for unloading. This probably would yield a smaller long-time horsepower output for a human than the 0.45 value for a rotary cycle.[1]

The efficiency is the ratio of useful to total work

$$\eta = \frac{\int_0^T F(t)\dot{x}\,dt}{\int_0^T \left| F(t)\ \ (\dot{x}-\dot{y}) \right| dt} = -\frac{\int_0^T \ddot{y}\dot{x}\,dt}{\int_0^T \left| (g-\ddot{y})\ \ (\dot{x}-\dot{y}) \right| dt} \tag{8}$$

This expression is evaluated analytically for sine wave forcing and by analog simulation for sine, square, and triangle forcing.

Analysis of Spring-Dashpot System

Consider the system of Fig. 1 where both spring and dashpot external forces act. A harmonic forcing function $f(t) = \sin\omega t$ is chosen. The oscillatory, steady-state solutions of Eq. (1) and (2) are desired. Transients and static deflections are neglected. The solution is simply

$$x(t) = X\sin(\omega t - \phi) \tag{9}$$

and

$$y(t) = Y\sin\omega t \tag{10}$$

where

$$Xk/F = 1/\{[1-(\omega/\omega_n)^2]^2 + (2\zeta\omega/\omega_n)^2\}^{1/2} \tag{11}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right) \quad 0 \leq \phi \leq 180° \tag{12}$$

$$Ym\omega^2/F = 1 \tag{13}$$

The cage motion $x(t)$ lags the force $f(t)$ by angle ϕ, and the human c.g. $y(t)$ is in phase with the force [by virtue of the opposing sign conventions chosen for $F(t)$ and y(t)].

A nondimensional leg extension is defined

$$(x-y)/X \equiv B\sin(\omega t - \alpha) \tag{14}$$

where α is the phase angle by which the leg extension $x-y$ lags the force. This angle plays a very important role in the efficiency results. From Eq. (8-12), one finds

$$B = [\sin^2\phi + (\cos\phi - Y/X)^2]^{1/2} \tag{15}$$

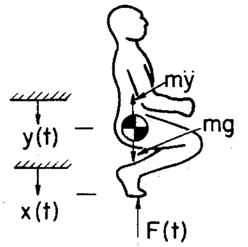

Fig. 2 Equilibrium of human.

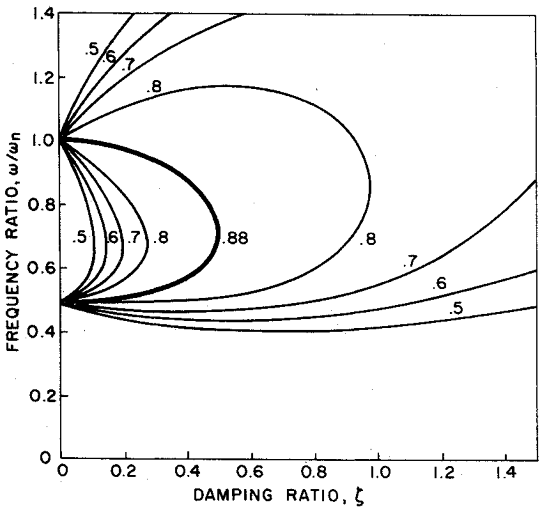

Fig. 3 Efficiency contours for sinusoidal loading ($\epsilon=0$, $F/mg=1.0$, $M/m=0.3333$); contours omitted for $\eta<0.50$.

$$\alpha=\tan^{-1}\frac{\sin\phi}{\cos\phi-Y/X} \qquad 0\leq\alpha\leq180° \tag{16}$$

$$Y/X=(\omega_n/\omega)^2(M/m)\ \{[1-(\omega/\omega_n)^2]^2+(2\zeta\omega/\omega_n)^2\}^{1/2} \tag{17}$$

By use of the preceding dynamic results, one calculates the useful work per cycle

$$(W_{useful}/FX)=\pi\sin\phi \tag{18}$$

By limiting the forcing to $F/mg\leq1.0$, the total human work per cycle consists of only one loading and one unloading portion, and becomes

$$(W_{hum}/mgX)=2(1+\epsilon)B+(\pi/2)$$
$$(1-\epsilon)B(F/mg)\sin\alpha \tag{19}$$

The sinusoidal cycle efficiency for $F/mg\leq1.0$ is, hence

$$\eta=W_{useful}/W_{hum} \tag{20}$$

$$\eta=\frac{\pi(F/mg)\sin\phi}{B[2(1+\epsilon)+(\pi/2)(1-\epsilon)(F/mg)\sin\alpha]} \tag{21}$$

which can be viewed as

$$\eta=\text{function}[\omega/\omega_n,\zeta,F/mg,M/m,\epsilon,f(t)] \tag{22}$$

The useful power developed by the human is

$$P_{useful}=\tfrac{1}{2}FX\omega\sin\phi \tag{23}$$

and the half stroke S is defined using Eq. (14)

$$S=XB \tag{24}$$

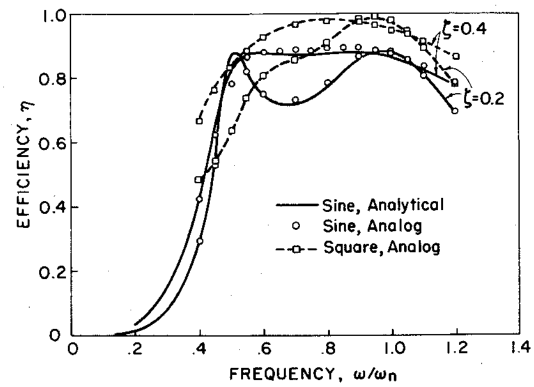

Fig. 4 Frequency sweep: $\zeta=0.2$ and 0.4, $F/mg=1.0$, $\epsilon=0$, $M/m=0.3333$.

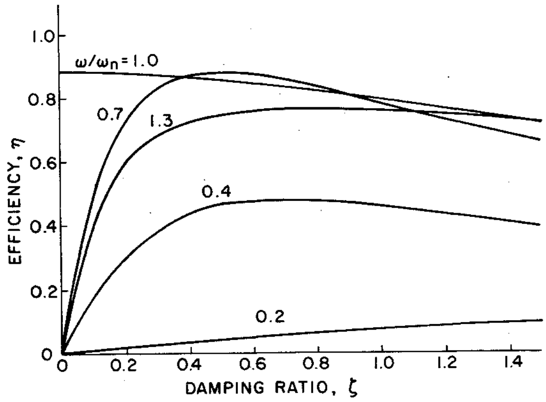

Fig. 5 Typical variation of efficiency with damping ($F/mg=1.0$, $\epsilon=0$, $M/m=0.333$, sine wave force).

Numerical Results for the Spring-Dashpot System

In spite of the simplicity of the dynamical system, the efficiency expression [Eq. (22)] shows complicated dependence on some of its parameters, notably ω/ω_n and ζ. The "switching" behavior in the denominator causes the interesting results.

A. Reference Case

Results will be presented in detail for a reference, or baseline, case. This case will bring out the complexity of the ω/ω_n and ζ dependence. The more moderate effects of F/mg, M/m, ϵ, and $f(t)$ will be discussed later. The reference case is

$$\eta=\text{function}(\omega/\omega_n,\zeta,1.0,0.3333,0.0,\sin\omega t) \tag{25}$$

The force ratio F/mg is unity, which is the highest value possible without causing negative g forces on the pilot and requiring toe straps. The mass ratio $M/m=0.3333$ is chosen as a typical for a 150-lb man flying a 50-lb ultralight aircraft. The biological penalty factor is set at zero.

Figure 3 is a contour chart showing constant elevation lines of efficiency. The maximum efficiency reached for this baseline case is 88%, occurring on a horseshoe shaped curve. For a given man/platform configuration at low damping, there are two frequencies for which efficiency is maximum. The lower intercept of this curve on the ordinate corresponds to resonance of the man and platform, locked together and moving on the spring support at $\omega=[k/(M+m)]^{1/2}$. The upper intercept is the resonance of the cage alone on the support at $\omega=(k/M)^{1/2}$. (Later calculations will show that the upper branch of the horseshoe is not useful for a human in a ultralight aircraft.) The phase angle α is 90° at all points on this optimum efficiency horseshoe. This is the phasing at which the man does the most total work on the platform and gravity.

Elevation contours for less than 0.50 efficiency are not given in this figure.

In order to study the reference case further, cuts of the efficiency surface will be made parallel to the frequency and damping axes. Figure 4 shows frequency "sweeps" and contains some analog data (the points) as well as analytical results. The analog data confirm the nature of the horseshoe curve for the sine wave exitation. One also sees that the square wave exitation is more efficient over most of the operating range. The case of $\zeta = 0.4$ shows a rather high, broad plateau of efficiency for sinusoidal exitation which would provide desirable operating characteristics. Figure 5 gives cuts of the efficiency surface parallel to the damping axis.

B. General Case

Having developed some intuition about the efficiency surface for the reference case, one now can carry out parameter studies. Figure 6 indicates that the square wave is typically the most efficient forcing function, with sine wave next, and triangular wave least efficient. Figure 6 also shows that large F/mg causes high efficiency. This means that, as the unsteady force in the human's legs tends to dominate the gravitational bias, the cycle becomes more efficient. Gravity must be viewed as the loss factor. If this cyclic process were done in a horizontal plane, one would have $\eta \to 1.0$ as $F/mg \to \infty$, at least for the sine wave. (The square wave may not reach this limit.)

The biological penalty factor ϵ is studied in Fig. 7. It has an adverse affect on sine, square, and triangular (not shown) cases, with a monotonic decrease in efficiency as ϵ is raised. The remaining parameter is mass ratio M/m. It will be studied through its effect on the optimum efficiency contour, in the next section.

C. Optimum "Horseshoe"

Numerical results have shown optimum efficiency to occur where $\alpha = 90°$. Since

$$\alpha = \tan^{-1} \frac{\sin\phi}{\cos\phi - Y/X}$$

this occurs at

$$\cos\phi = Y/X \tag{26}$$

and the maximum efficiency reached is

$$\eta_{max} = \frac{\pi(F/mg)}{2(1+\epsilon) + (1-\epsilon)(\pi/2)(F/mg)} \tag{27}$$

This η_{max} is not a function of ω/ω_n, ζ or M/m. For example, the value of η_{max} for the reference case (Fig. 3) is

$$\eta_{max} = \pi/(2+\pi/2) = 0.87988 \tag{28}$$

It was found earlier that the optimum value of α, and hence η_{max}, was obtained on the horseshoe-shaped curve in Fig. 3. If the mass ratio is changed, the same $\eta_{max} = 0.87988$ is obtainable, but on a different frequency horseshoe. Figure 8 consists of a family of horseshoe curves as would be found from efficiency elevation charts for different mass ratios.

Finally, η_{max} from Eq. (27) is plotted in Fig. 9. This gives the maximum efficiency possible for all sinusoidal forcing without toe straps. These values may or may not be obtainable dynamically for specific values of ω/ω_n, ζ, and M/m.

D. Scaling of System to Human Capabilities

For a human, the half stroke S has an upper limit of roughly 1.5 ft, and any cycle requiring motion greater than this is unrealizable. Also, the human is limited to power out-

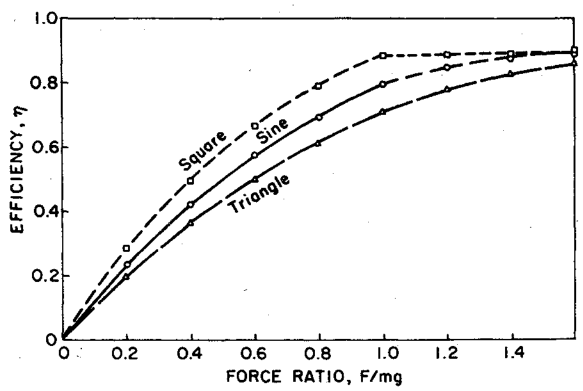

Fig. 6 Comparison of sinusoidal, square, and triangular forcing functions ($\epsilon = 0$, $\delta = 1.0$, $\omega/\omega_n = 1$, $M/m = 0.3333$); analog computer results.

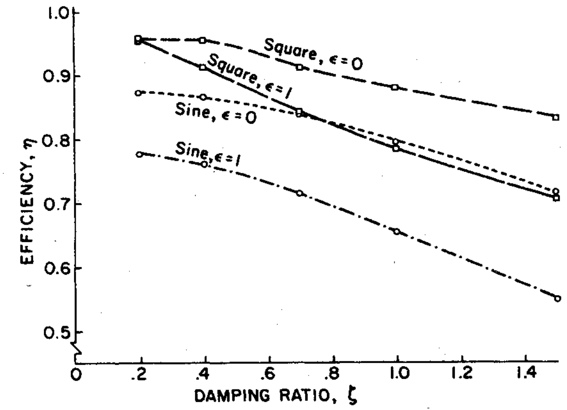

Fig. 7 Effect of penalty factor ϵ on square and sine wave cases; $\omega/\omega_n = 1.0$, $M/m = 0.3333$, $F/mg = 1.0$; analog data.

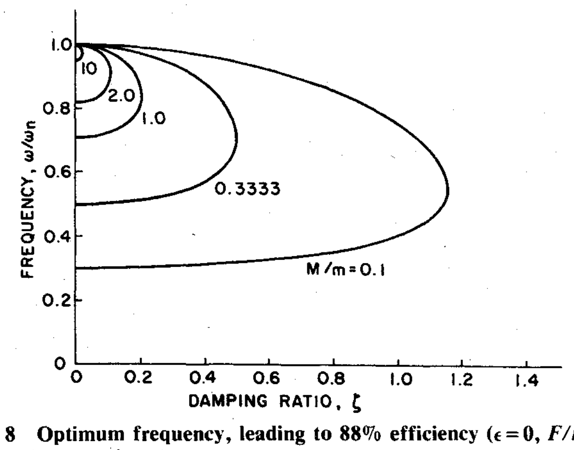

Fig. 8 Optimum frequency, leading to 88% efficiency ($\epsilon = 0$, $F/mg = 1.0$, sine wave force).

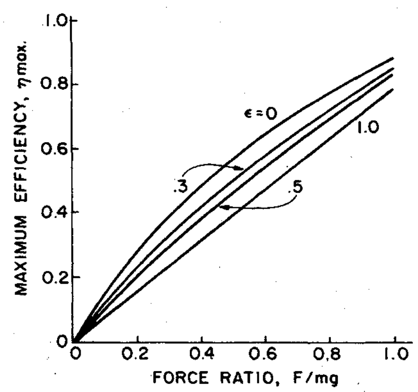

Fig. 9 Maximum efficiency kinematically possible.

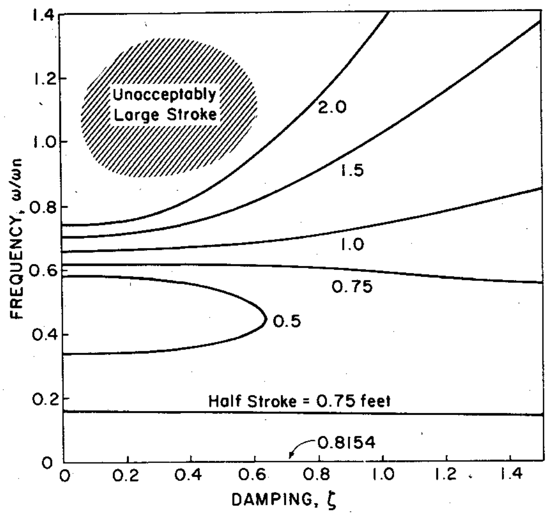

Fig. 10 Stroke contours for sinusoidal loading ($\epsilon=0$, $F/mg=1.0$, $M/m=0.3333$).

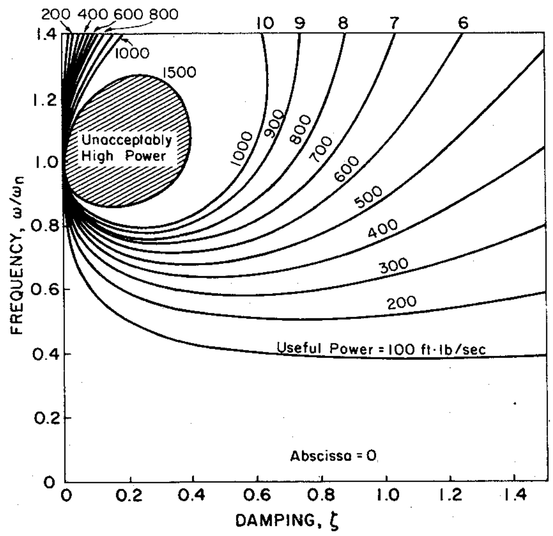

Fig. 11 Power contours for sinusoidal loading ($\epsilon=0$, $F/mg=1.0$, $M/m=0.3333$).

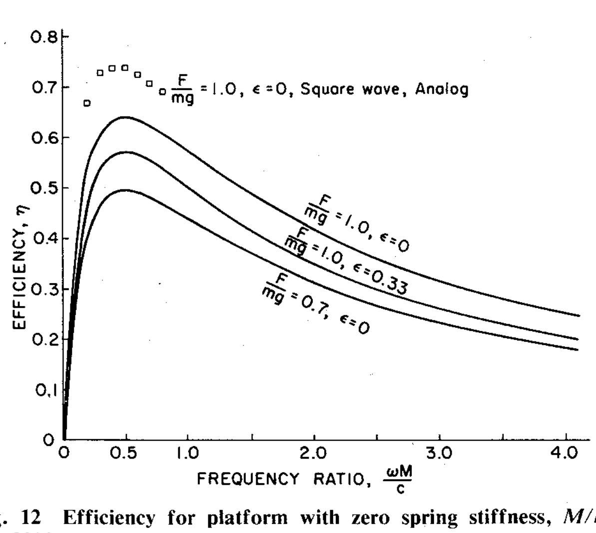

Fig. 12 Efficiency for platform with zero spring stiffness, $M/m=0.3333$.

put of less than 1500 ft-lb/sec, even for short periods of time.[1] Again, cycles calling for more power than this are unattainable.

Assume a human weight of 150 lb, and a frequency of operation of 1 Hz. Power generation and stroke required are

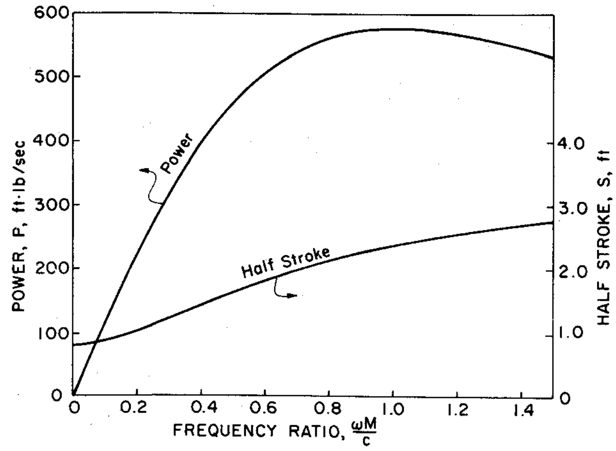

Fig. 13 Useful power and half stroke for human on platform with zero spring stiffness $M/m=0.333$, $F/mg=1.0$.

calculated for the reference case (Figs. 10 and 11). Useful power generated is presented, rather than total human power. The area near $\omega/\omega_n=1.0$ and $\zeta=0$ is a forbidden area for reasons of both power and stroke required. Combining results of Figs. 3, 10, and 11, one can see that, for continuous power output in the half-horsepower range, the human/platform system with spring-damper suspension should operate in the vicinity of $\zeta=0.4$ and $\omega/\omega_n=0.55$-0.70. This also allows a range of horsepower and stroke at efficiencies near 88%. One could better this efficiency only by use of toe straps (and negative g loading) while trying to excite the platform with a square wave leg force, both of which would be uncomfortable over a period of time.

Limiting Case – Platform with Spring Removed

Thus far, the platform has been suspended on a spring and dashpot. If the spring is removed and the system is reanalyzed, one less dimensionless ratio is needed, because one characteristic time $(M/k)^{1/2}$ has been lost. This limiting case is studied to show the importance of the spring to the system. The choice of $f(t)=\sin(\omega t)$ leads to the oscillatory solution

$$x(t)=X\sin(\omega t-\phi)y(t)=Y\sin\omega t$$

$$\frac{X\omega c}{F}=\frac{1}{[(\omega M/c)^2+1]^{1/2}}$$

$$\phi=\tan^{-1}\frac{1}{-(\omega M/c)} \qquad 90° \le \phi \le 180°$$

$$Ym\omega^2/F=1$$

$$Y/X=(M/m)[1+(c/M\omega)^2]^{1/2}$$

and the expressions for B, α, and η remain as in Eq. (15, 16, and 21) Now we have

$$\eta=\text{function}[\omega M/c, F/mg, M/m, \epsilon, f(t)]$$

A. Reference Case

The solution in terms of dimensionless ratios is given in Fig. 12. Again, a mass ratio typical of a human and an ultralight aircraft is assumed. The maximum efficiency gained for harmonic oscillation, without toe straps, is 0.64. Square wave exitation, also at $F/mg=1.0$, has a peak efficiency of 0.74. [Eq. (27), for maximum efficiency, does not hold for the zero spring case, because the optimum phase lag of $\alpha=90°$ is unobtainable dynamically.] These peak efficiencies vary somewhat with mass ratio. Efficiency could be raised slightly with toe straps, but the penalty factor ϵ would lower it.

B. Zero Spring Platform Scaled to Human Capabilities

Again, choose a human weighing 150 lb and oscillating at 1 Hz (Fig. 13). The system has some problems with the impendance match between human and platform. The suspension is too "soft" for the human. In order to operate with peak efficiency at $\omega M/c=0.5$, the human would have to use a half stroke of 1.65 ft, which is not attainable. This means operating at smaller stroke with less efficiency. Nevertheless, the cycle does useful work, and graphs such as Fig. 12 and 13 can be used to optimize performance for the individual involved. The best opeating frequency is $\omega=0.5\ c/M$, and so the numerical value of the equivalent viscous damping coefficient c should be found early in the design cycle.

Conculsions

Efficiency calculations show losses in the vertical work cycle; these are caused by gravity and the inability of the human to store energy during unloading portions of the cycle. The efficiency depends on the operating frequency and the damping of the platform suspension. Increasing the force ratio F/mg increases efficiency. Square wave excitation is, in alsmost all cases, more efficient than sine wave, and sine wave is always better than triangular excitation.

At optimum efficiency, the leg force $f(t)$ leads the leg extension $x(t)-y(t)$ by 90°. For a given cage suspension with low damping ratio ζ, this condition is met at two frequencies. The lower corresponds to the resonance of human and platform oscillating in phase on the spring. The higher corresponds to the platform alone, resonating on the spring. Operation at the higher frequency is not a useful mode because of the large stroke and power requirements. If the spring is removed from the suspension, neither of these resonant frequencies is attainable, causing inefficiency.

A spring-dashpot-supported platform can be operated at up to 88% efficiency in a realistic situation using sinusoidal forcing and no toe straps. The comparable maximum when the spring is removed is 64%. If the human is to be penalized for the tiring effect of the unloading portion of the cycle, both of these numbers will be lowered.

A better impedance match clearly is obtained between man and machine when spring-like forces oppose his motion. For a rigid aircraft, these can be provided by airplane aerodynamics. Another means is to introduce spring forces through spring-loaded wings or spring-suspended pilot harnesses. A recent paper by Wolf[4] proceedes in the latter direction and follows a type of spring-winged airplane by Lippisch.[5] Now it should be appreciated that spring-like forces are important to the system, and careful matching of man and machine is needed to deliver high efficiency in vertical pumping motions.

References

[1] Wilkie, D.., "Man as an Aero Engine," *Journal of the Royal Aeronautical Society*, Vol. 64, Aug. 1960, pp. 447-481.

[2] Nonweiler, T.R.F., "The Man-Powered Aircraft," *Journal of the Royal Aeronautical Society*, Vol. 62, Oct. 1965, pp. 723-734.

[3] Smith, G., "Pseudo-Ornithopter Propulsion," *First International Symposium on the Technology and Science of Low Speed and Motorless Flight*, The Soaring Society of America, Boston Mass., Oct. 1972, pp. 395-398.

[4] Wolf, J., "The Technological Prospects for Oscillating-Wing Propulsion of Ultralight Gliders," *Proceedings of the 2nd International Symposium on the Technology and Science of Low-Speed and Motorless Flight*, The Soaring Society of America, Boston Mass. Sept. 1974.

[5] Lippisch, A.M., "Man Powered Flight in 1929," *Journal of the Royal Aeronautical Society*, Vol. 64, July 1960.